

All uppercase letter variables are positive integers unless otherwise stated. All fractions containing uppercase letter variables are in lowest terms. NOTA means "None of the Above."

- 1) Mettaton picks a random number θ from the interval $[0, 2\pi]$. Undyne gives the value of $\cos 2\theta$, while Alphys gives the value of $\cos \theta - \frac{1}{2}$. If the probability that Undyne's value is greater than Alphys's value is equal to $\frac{A}{B}$, find $A + B$.

Hint: To avoid confusion with the fraction, read the instructions.

- (A) 3 (C) 6 (E) NOTA
(B) 4 (D) 7
- 2) Brill the ant is climbing out of a well that is 50 meters deep. Every day, she climbs up 5 meters, but in the night when she sleeps, she slips down 2 meters. How many days does it take Brill to get out of the well if she starts from the bottom?
- (A) 15 (C) 17 (E) NOTA
(B) 16 (D) 18
- 3) Rik is making dilute hydrochloric acid (HCl) in his laboratory. He begins with 24 liters of 30% HCl. How much 5% HCl must he add this to obtain a solution that is 10% HCl?
- (A) 72 liters (C) 120 liters (E) NOTA
(B) 96 liters (D) 144 liters
- 4) Sanjoy has class at 3:35. It is now 3:30, so Sanjoy decides to calculate the angle between the hour and the minute hand at 3:35. What angle does Sanjoy correctly come up with?
- (A) 102.5° (C) 112.5° (E) NOTA
(B) 105° (D) 120°
- 5) Trevor bikes 2 miles north, then 3 miles west, then 7 miles south, and then 9 miles west. Find the positive difference between the distance Trevor travels and his distance from his original starting point.
- (A) 0 miles (C) 7 miles (E) NOTA
(B) 6 miles (D) 8 miles
- 6) 10% of the residents of Fiore are nerds. A test can detect someone's being a nerd with 98% accuracy, as well as detecting someone's not being a nerd with 97% accuracy. Given that the test says Natsu is a nerd, what is the probability that Natsu is in fact a nerd?
- (A) 0.576 (C) 0.784 (E) NOTA
(B) 0.640 (D) 0.980
- 7) The number $-i$ has six roots in the Argand Plane. These roots can be expressed in the form $z_n = \cos \theta_n + i \sin \theta_n$ for $n \in \{1, \dots, 6\}$ and distinct $\theta_n \in [0, 2\pi)$. Find $\sum_{n=1}^6 \theta_n$.
- (A) 4π (C) $\frac{11\pi}{2}$ (D) $\frac{13\pi}{2}$
(B) $\frac{9\pi}{2}$ (E) NOTA

- 8) Bjergsen is running through the jungle at a rate of 9 meters per second. 2 seconds after he passes Svenskeren, Svenskeren runs after him at a rate of 12 meters per second. How many meters will Svenskeren run from the time Bjergsen passes him before he catches Bjergsen?
- (A) 54 (C) 81 (E) NOTA
(B) 72 (D) 108
- 9) Let the answer to this question be equal to $n^2 + 5n + 4$, where n is the answer to this question. Find the value of n .
- (A) -2 (C) -1 (E) NOTA
(B) 0 (D) Impossible to determine
- 10) Waffles the Dolphin swims in a river with a constant current. It takes him 30 minutes to swim 10 miles upstream, and 10 minutes to swim the 10 miles back. Find the current of the river.
- (A) 10 mph (C) 20 mph (E) NOTA
(B) 15 mph (D) 40 mph
- 11) Find the length of the domain of the function $f(x) = \ln(9 - x) - \sqrt{\frac{x^2 - 6x - 16}{3x - 2x}}$.
- (A) 3 (C) 11 (E) NOTA
(B) 7 (D) Infinite
- 12) The amount of scientific power Eridan has t hours after waking up at 9:00 PM is modeled by the function $s(t) = 413 - 100t + 25t^2$. When is Eridan's scientific power minimized?
- (A) 9:30 PM (C) 10:30 PM (E) NOTA
(B) 10:00 PM (D) 11:00 PM
- 13) Find the sum of the entries of $\begin{bmatrix} -3 & 5 & -5 & -4 & 2 \\ 0 & -2 & 1 & 2 & 0 \\ -2 & 0 & 3 & 4 & -1 \\ 1 & -3 & 1 & 3 & 5 \\ 5 & -1 & 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 3 & 1 & 3 & -2 & -4 \\ 5 & -4 & 1 & 3 & -1 \\ 0 & 2 & 0 & -2 & 4 \\ -1 & 4 & -5 & -5 & 0 \\ 4 & 0 & 3 & -3 & -2 \end{bmatrix}$.
- (A) -34 (C) -31 (E) NOTA
(B) -33 (D) -30
- 14) Tom Sawyer can paint a fence in 8 hours. If Tom is joined by Huck Finn, they can paint the fence in 2 hours. If Tom is joined by Joe Harper, they can paint the fence in 6 hours. How long would it take Huck Finn and Joe Harper to paint the fence?
- (A) 144 minutes (C) 160 minutes (E) NOTA
(B) 150 minutes (D) 162 minutes
- 15) Find the area enclosed by the ellipse $4x^2 - 4xy + 7y^2 = 24$.
- (A) $\frac{12\pi\sqrt{42}}{7}$ (C) $6\pi\sqrt{10}$ (E) NOTA
(B) $2\pi\sqrt{6}$ (D) $\frac{84\pi\sqrt{3}}{5}$

- 16) Emily and RJ are playing a game involving the flipping of an unfair coin. They take turns flipping the coin, Emily flipping it twice, followed by RJ flipping it once. Emily wins if she flips two heads on the same turn before RJ flips a head (otherwise RJ wins). If Emily goes first, they have an equal chance of winning. If RJ goes first, what is the probability that he wins?
- (A) $\frac{1}{2}$ (D) $\frac{\phi}{2}$
 (B) $\phi - 1$ (E) NOTA
 (C) $\frac{1+\sqrt{2}}{3}$
- 17) It is given that some numbers a, b, c, d satisfy the equation $a^2 + b^2 = c^2 + d^2$. Find the square of the maximum value of $\frac{20bc+18bd+20ad-18ac}{a^2+b^2+c^2+d^2}$.
- (A) 181 (C) 216 (E) NOTA
 (B) 204 (D) 322
- 18) Saaketh is selling food at the MAØ bake sale. He sells brownies for \$1.50 each and cookies for \$0.50 each. If he sells 49 items and makes \$52.50, how many cookies did he sell?
- (A) 16 (C) 19 (E) NOTA
 (B) 18 (D) 21
- 19) Steven is walking in the coordinate plane from his dorm to the library. However, he cannot walk directly to the library, since there is a fountain in the way. Thus, he decides to walk in a straight line from his dorm to a certain point, turn, and walk in a straight line to the library. He notices that no matter where he turns, he will always walk the same total distance to the library. The set of points where Steven can walk to and turn forms which of the following figures?
- (A) Ellipse (C) Parabola (E) NOTA
 (B) Hyperbola (D) Impossible to determine
- 20) Captain Kirk is walking on the bridge of the Starship Enterprise. On the i th step of his walk, he moves $\frac{1}{2^i}$ meters forward, and then turns 60 degrees to the left. Given that his first step is from the origin to $(\frac{1}{2}, 0)$, the point Captain Kirk will approach as i increases without bound is $(\frac{A}{B}, \frac{\sqrt{C}}{D})$ for square-free C . Find $A + B + C + D$.
- (A) 9 (C) 12 (E) NOTA
 (B) 10 (D) 14
- 21) $n!$ ends in 2018 consecutive zeroes. Find the sum of the digits of the smallest integer value of n .
- (A) 16 (D) 24
 (B) 18 (E) NOTA
 (C) 21

22) The Arcturan Starfunction $*$ (n) counts the number of ordered pairs of integers (a, b) such that $|a + bi| \leq n$. Find $*$ ($\sqrt{26}$).

- (A) 79 (C) 89 (E) NOTA
(B) 81 (D) 91

23) Old Woman Josie does not like numbers that contain zeroes. However, she does like numbers whose digits sum to 8, as long as they contain no zeroes. She feels neutral about all other numbers. How many positive integers does Old Woman Josie like?

- (A) 128 (C) 223 (E) NOTA
(B) 161 (D) 279

24) Couper's favorite trigonometric function is the ExodiaTrig function, which he defines as $ET(x) = \sin x + \cos x + \tan x + \cot x + \sec x + \csc x$. Find $ET(120)$.

- (A) $\frac{-15-\sqrt{3}}{6}$ (C) $\frac{5-5\sqrt{3}}{2}$ (E) NOTA
(B) $\frac{-15+\sqrt{3}}{6}$ (D) $\frac{5+5\sqrt{3}}{2}$

25) A recursion is defined by the following:

$$\begin{aligned}x_{n+1} &= 2x_n + 3y_n \\ y_{n+1} &= x_n + 2y_n\end{aligned}$$

Given that $x_0 = 2$ and $y_0 = 1$, then the following formula is true, given that the value of C is square-free.

$$x_n = \frac{(A + B\sqrt{C})^{n+1} + (A - B\sqrt{C})^{n+1}}{D}$$

Find $x_A + y_B + x_C + y_D$.

- (A) 93 (C) 142 (E) NOTA
(B) 104 (D) 175

26) Radleigh's radness in a given year Y is equal to the following:

$$\sum_{n=1}^Y n^{n+3}$$

Find the units digit of Radleigh's radness in the year 2018.

- (A) 3 (C) 7 (E) NOTA
(B) 4 (D) 9

27) Broder rides his motorcycle off into the sunset along a parabola $f(x)$. It is given that $f(2015) = f(2016) = 2$ and $f(2017) = 1$. Find the value of $f(2020)$.

- (A) -10 (C) -8 (E) NOTA
(B) -9 (D) -7

- 28) Let the number of ordered triplets of positive integers (a, b, c) such that the following equation is satisfied be equal to K . Find the sum of the digits of K .

$$\sum_{i=0}^a (b + ci) = 2018$$

Hint: You may use the information that 1009 is prime.

- (A) 17 (C) 13 (E) NOTA
(B) 15 (D) 12

- 29) In calculus, the *derivative* of a function at a point is defined as the slope of the tangent line to that function at that point. A derivative is calculated with the difference quotient:

$$\frac{df(x)}{dx} = \lim_{n \rightarrow 0} \frac{f(x+n) - f(x)}{n}$$

Find the derivative of $\cos x$ with respect to x .

- (A) $\sin x$ (C) $-\sin x$ (E) NOTA
(B) $\cos x$ (D) $-\cos x$

- 30) It takes an orchestra of 150 members 60 minutes to play Rex Duodecim Angelus at a tempo of 120 beats per minute. How long would it take an orchestra of 50 members to play Rex Duodecim Angelus at 120 beats per minute?

- (A) 20 minutes (C) 120 minutes (E) NOTA
(B) 60 minutes (D) 180 minutes